FST Semester 1 Review

Find the distance between the following points
1. (-3, 4) and (2,1)
   \[ d = \sqrt{(-3-2)^2 + (4-1)^2} = \sqrt{5^2 + 3^2} \approx 5.83 \]

2. (3,3) and (-5,9)
   \[ d = \sqrt{(3+5)^2 + (3-9)^2} = \sqrt{8^2 + 6^2} \approx 10 \]

Simplify the following expressions
3. \[ \frac{3^4 m^{-1} n^3}{9^2 m^{-1} n^{-1}} \] No Negative Exps.
   \[ \frac{4^n}{m^3} \]

4. \[ \frac{(3^2 u^3 v^{-3})^{-2}}{9 u^{-2} v^2} = \frac{u^2 v^6}{9^3 u^6 v^2} \]
   \[ = \frac{V^4}{729 u^4} \]

Rationalize the denominator
5. \[ \frac{2}{4 - \sqrt{2}} \]
   \[ \frac{4 + \sqrt{2}}{16 - 2} \]
   \[ \frac{4 + \sqrt{2}}{14} \]

6. \[ \frac{1}{2 - \sqrt{3}} \]
   \[ \frac{2 + \sqrt{3}}{4} \]
   \[ \frac{2 + \sqrt{3}}{2 + \sqrt{3}} \]

Perform the indicated operation of the polynomials
7. \[ -(3x^2 + 2x) + (1 - 5x) \]
   \[ -3x^2 - 7x + 1 \]

8. \[ 8y - [2y^2 - (3y - 8)] \]
   \[ 8y - 2y^2 + 3y - 8 \]
   \[ -2y^2 + 11y - 8 \]

Factor Completely
9. \[ 3x^2 + 14x + 8 \] "5 step"
   \[ 3x^2 + 12x + 2x + 8 \]
   \[ 3x(x+4) + 2(x+4) \]
   \[ (x+4)(3x+2) \]

10. \[ 2x^4 + 21x^2 + 10 \] "grouping"
    \[ 2x^4 + x^2 + 20x^2 + 10 \]
    \[ x^2(2x^2+1) + 10(2x^2+1) \]
    \[ (2x^2+1)(x^2+10) \]
Perform the operation and Simplify

12. \( \frac{3x}{x+2} - \frac{4x^2 - 5}{2x^2 + 3x - 2} \)

13. \( \frac{4x - 6}{(x-1)^2} + \frac{2x^2 - 3x}{x^2 + 2x - 3} \)

Solve the following equations

15. \( \frac{5}{x-2} = \frac{13}{2x-3} \)

17. \( \sqrt{2x+3} + \sqrt{x} = 2 \)

Solve the given equation for the variable

19. Solve for \( r \): \( V = \frac{1}{3} \pi r^2 h \)

Solve the following inequality

21. \(-19 < 3x - 17 \leq 34\)

22. \( |x - 2| < 1\)
Perform the operation and write the result in standard form

23. \( (10 - 8i)(2 - 3i) \)
\[
= 20 - 30i - 16i + 24c^2 / -24
\]
\[-4 - 46i
\]

24. \( \frac{4(1+i)}{2-3i} + \frac{2}{1+i} \)
\[
= \frac{4 + 4i + 4 - 2i}{2-3i}(1+i)
\]
\[\frac{8 - 2i}{2-3i}(1+i)
\]

Word Problems: Simple interest and complex interest

25. \( \text{P dollars invested at an interest rate } r \text{ compounded annually increases to an amount } A = P(1 + r)^2 \text{ in 2 years.} \) If an investment of $5000 is to increase to an amount greater than $5500 in 2 years, then the interest rate must be greater than what percent?

\[
\sqrt{\frac{5500}{5000}} = \sqrt{(1 + r)^2}
\]

0.1 > r

26. A biologist introduces 200 ladybugs into a crop field. The population \( P \) of the ladybugs is approximated by the model \( P = \frac{1000(1 + 3t)}{5 + t} \) where \( t \) is the time in days. Find the time required for the population to increase to at least 2000 ladybugs.

\[
1000 = \frac{1000(1 + 3t)}{5 + t}
\]

\[
t = \frac{1000}{1000} - 2000 = \frac{1000}{5 + t} - 2000
\]

\[
t = \frac{9000}{1000}
\]

It takes at least 9 days.

27. Given the points what is the slope of the line.

\( A(-1,2) \) \( B(3,4) \)

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2}{4} = \frac{1}{2}
\]

Find the equation of the line given the points

28. \( C(4,1) \) \( D(-5,-5) \)

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6}{9} = \frac{2}{3}
\]

29. \( A(-1,2) \) \( B(3,4) \)

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 2}{3 - (-1)} = \frac{1}{2}
\]

30. \( C(4,1) \) \( D(-5,-5) \)

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9}{10} = \frac{9}{5}
\]

31. Given the equation of the line, \( y = \frac{2}{3}x - 8 \), find the equation of the line parallel to it and through the point (1,1)

\[
l = \frac{3}{2}(1) + b
\]

\[
y = \frac{2}{3}x - \frac{5}{3}
\]

32. Given the equation of the line, \( y = \frac{1}{3}x + 3 \), find the equation of the line perpendicular to it and through the point (2,1)

\[
m = \frac{1}{3}
\]

\[
y = \frac{1}{3}x + b
\]

\[
y = 3x - 5
\]
33. Graph the four lines found in 29–32

34. \( f(x) = \sqrt{25 - x^2} \)

\[
\begin{align*}
15 - x^2 & \geq 0 \\
\sqrt{15} & \geq x \quad (5+x)(5-x) \geq 0 \\
5 & \geq x \\
5 & \leq x \leq 5
\end{align*}
\]

The domain of \( f(x) \) is \( \{ x \mid -2 \leq x \leq 0 \} \).

Given the following function \( f(x) = \begin{cases} 
5 & -2 \leq x \leq 0 \\
8x - 5 & x > 0
\end{cases} \)

evaluate it for the given values:

36. \( f(-1) \quad \text{middle} \) 37. \( f(4) \quad \text{bottom} \) 38. \( f(-3) \quad \text{top} \)

\( f(-1) = 5 \) \( f(4) = 37 \) \( f(-3) = 27 \)

39. Graph the following function \( f(x) = \begin{cases} 
5 & -2 \leq x \leq 0 \\
8x - 5 & x > 0
\end{cases} \)
Describe the transformation of the function

40. \( f(x) = (x - 2)^2 + 2 \)
   - Right 3
   - Up 2

41. \( f(x) = |x + \frac{3}{4}| - 5 \)
   - Left 3
   - Down 5

42. \( f(x) = -\sqrt{x + 1} + 4 \)
   - Reflect over x-axis
   - Left 1
   - Up 4

43. \( f(x) = (-x - 1)^3 - 3 \)
   - Reflect over x-axis
   - Up 1
   - Down 3

44. Graph the four functions from 40-43 (\#41, \#42, \#43, \#46)
Write the equation of the function.

45. A parabola moves up 3 and to the left 4 units

\[ y = a(x-h)^2 + k \]

\[ y = a(x+4)^2 + 3 \]

46. An absolute value graph is flipped on the x-axis and the new vertex is located at (2, 3)

\[ y = a|x-h| + k \]

\[ y = -a|x+2| + 3 \]

Given the functions \( f(x) = 3 - 2x \) and \( g(x) = \sqrt{x} \), perform the following operations.

47. \( (f+g)(x) \)

\[ 3 - 2x + \sqrt{x} \]

\[ (f+g)(x) = -2x + \sqrt{x} + 3 \]

48. \( (fg)(x) \), multiplying

\[ (3 - 2x)(\sqrt{x}) \]

\[ = 3\sqrt{x} - 2x \sqrt{x} \]

49. \( f(g(x)) \), "g into f" [composition]

\[ = 3 - 2\sqrt{x} \]

\[ = 3 - 2\sqrt{x} \]

50. \( (g \circ f)(x) \), "f into g" [composition]

\[ = 3 - 2\sqrt{x} \]

Given the function, find the inverse and check your answers.

51. \( f(x) = \frac{1}{2}x - 3 \)

\[ y = \frac{1}{2}x - 3 \]

\[ x = \frac{1}{2}y + 3 \]

\[ \frac{1}{2}y = x - 3 \]

\[ 2(y + 3) = x \]

Find the x and y intercept of the following function.

53. \( y = (x + 1)^2 \)

\[ y = (0 + 1)^2 \]

\[ y = 1^2 = 1 \]

\[ y = (0 + 1)^2 \]

\[ y = 0 \]

\[ x = -1 \]

54. \( y = |x - 4| - 4 \)

\[ y = |0 - 4| - 4 \]

\[ y = 4 - 4 \]

\[ x = 0 \]

\[ y = |x - 4| - 4 \]

\[ 4 = 1x - 4 \]

\[ x = 8 \]

\[ x = 0 \]
55. Given the graph of the function state the domain and range

\[ D: \text{All Real } \#s \]
\[ R: \text{All Real } \#s \geq -4 \]

56. Given Graph state the Domain, Range, Inc, Dec, Con, Min and Max

\[ D: -8 \leq x \leq -4 \text{ and } -4 \leq y \leq 0 \text{ and } 0 \leq x \leq 8 \]
\[ R: -9 \leq y \leq 2 \]

Increasing: \([-8, -4) \cup (0, 5]\]
Decreasing: \([5, 8]\]
Constant: \([-4, 0]\]
Graph the following functions.

57. \( h(x) = x^2 + 5x - 4 \)

\[ V(-2.5, -10.25) \]

58. \( y = (x - 3)^2 \)

Use long division or synthetic division to divide.

59. \( \frac{24x^2 - x - 8}{3x - 2} \)

\[ \frac{8x + 5}{3x - 2} \]

60. \( \frac{2x^3 - 19x^2 + 38x + 24}{x - 4} \)

Use the four key steps to graph the following equations.

61. \( f(x) = 2x^4 - 2x^2 - 40 \)

62. \( f(x) = 3x^2 - 12x + 3 \)

Use the Rational Zero Test to list all possible rational zeros of \( f \).

63. \( f(x) = -4x^3 + 8x^2 - 3x + 15 \)

64. \( f(x) = 3x^4 + 4x^3 - 5x^2 - 8 \)